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Interpretation(s) of elasticity in operant demand

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This brief report provides an account of varying interpretations of elasticity (η) in the operant demand framework. General references to "demand elasticity" have existed since the Exponential model of operant demand was proposed by Hursh and Silberberg (2008). This term has been used interchangeably with Essential Value (EV), P_{MAX}, and the rate of change constant α. This report provides an in-depth account of η and the various ways in which this metric has been used to interpret fitted demand functions. A review of relevant mathematic terms, operations associated with differentiating parameters, and worked solutions for η are provided for linear and nonlinear demand functions. The relations between η and EV, P_{MAX}, and α are described and explained in terms of their mathematical bases and recommendations are provided regarding their individual interpretation. This report concludes with recommendations for providing additional mathematical detail in published works and emphasizing a consistent use of terms when describing aspects of operant demand.

Key words: elasticity, operant demand, mathematical modeling, behavioral economics

The operant demand framework is increasingly used to evaluate relationships between reinforcers and the factors associated with their consumption (González-Roz et al., 2019; Hursh, 2000; Hursh & Roma, 2016; Kagel & Battalio, 1980; Strickland et al., 2020; Tidey et al., 2016; Zvorsky et al., 2019). Although the economic concept of demand has long existed within mainstream behavioral economics, the *operant* demand framework reviewed here is specific to an ecologically based perspective regarding human and nonhuman behavior, that is, reinforcer pathology rather than cognitive biases (Bickel et al., 2011). Specifically, the reinforcer pathology perspective holds that various forms of suboptimal choice are jointly driven by excessive valuations of particular reinforcers and a relative preference for immediate delivery of reinforcers, despite undesirable long-term effects (Bickel et al., 2014).

This approach and perspective have been applied broadly, with established utility in indexing substance abuse and misuse (Kaplan, Foster, et al., 2018; MacKillop et al., 2018) and the abuse liability for drugs (MacKillop et al., 2019; Strickland et al., 2020). Apart from substance use, this approach has also been used to evaluate how various forms of socially desirable behavior are affected by varying prices or levels of effort, for example, purchasing groceries (Foxall et al., 2010), "green" consumerism (Kaplan, Gelino, & Reed, 2018), and evaluating reinforcers in behavioral treatments (Gilroy et al., 2018).

The earliest applications of the operant demand framework emerged from reanalyses of experimental nonhuman research. Among the early researchers evaluating these principles from an ecological perspective, Lea (1978) provided an account of price elasticity of demand (η) in behavioral experiments.[1] Briefly, η (Greek letter eta; elasticity) is an expression of the relationship between changes in prices (P) and subsequent changes in consumption (Q) and η can be described in terms of *inelastic, elastic,* or *unit elastic* change. Quoting Lea (1978) on elasticity, "In an economic demand curve, elasticity of −1 means expenditure on the commodity is unaffected by price, whereas elasticity of absolute value less (more) than one means expenditure rises (falls) when price increases" (pg. 447). For convenience, η is illustrated across a range of inelastic, elastic, or unit elastic prices in Figure 1. Here, the left-hand plot shows a fitted demand curve and the right-hand plot illustrates the overall responding

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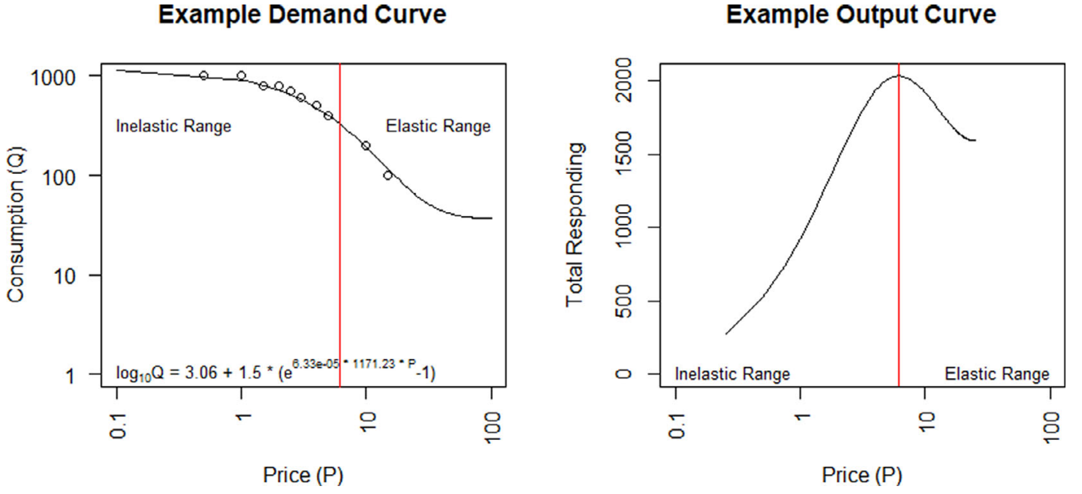
[1]It warrants noting that multiple forms of η exist, for example, demand, income. For the sake of this short report η will refer to price elasticity of demand, specifically, which may also be denoted as η_D or η_P.

Figure 1
Different Levels of Elasticity Across Prices



Note. The left plot illustrates inelastic $\eta < 1$, elastic $\eta > 1$, and unit elastic $\eta = 1$ demand and the right plot illustrate how inelastic demand is associated with increases in responding while the elastic range is associated with decreases in responding. [Color figure can be viewed atwileyonlinelibrary.com]

across prices. As noted by Lea (1978), prices in the inelastic range are associated with rising expenditure (i.e., increased responding) while prices in the elastic range are associated with decreasing expenditure. The P at which responding is at maximum is referred to as P_{MAX} (Hursh et al., 1987).

Although differential sensitivity to prices can be inferred visually (i.e., the peak of work output function), η has a specific mathematical basis and derivation. Within the rapid growth of the operant demand framework, some researchers have described η as a concept without presenting the specific mathematical basis for it and this has led to varying interpretations of η. For instance, it is our experience that some researchers erroneously presume individual rate parameters (e.g., α) or formulations of Essential Value (EV) are synonymous with η because, visually, each speaks to variability in how change is expressed in a curve. That is, higher α values are related to a greater sensitivity to price while lower α values are related to lesser sensitivity to price. Although several aspects of demand curve modeling speak to "sensitivity of price", referring to these all as "elasticity of demand" is not tenable because each of these measures represents a unique aspect of a demand function. The purpose of this report is to address each of the terms loosely referred to as "elasticity of demand" and to provide the mathematical basis for each and how they each relate to the operant demand framework.

Mathematical Terms

Demand Function

When we speak of *demand*, we refer to the degree to which some individual or organism will work to defend the bliss point consumption of a reinforcer. A demand *function* refers to some model or representation of the predicted level of demand for some reinforcer(s) as a function of one or more factors, for example, price, availability of alternatives. Although contemporary approaches in operant demand use nonlinear models to represent the demand function, see Hursh and Silberberg (2008) for a contemporary example, it warrants noting that most economists typically use linear models because multiple regression models can accommodate numerous variables apart from price alone (e.g., income, availability of substitutes). In linear models, η exists as a singular value and is either elastic, inelastic, or unit elastic (but remains the same across prices). Regardless of model, common terms used in demand functions include the number of goods consumed (Q) and price (P) per unit of consumption,

Figure 2
A Linear Demand Function Plotted in Log–Log Scales with a Constant η of – 10.5

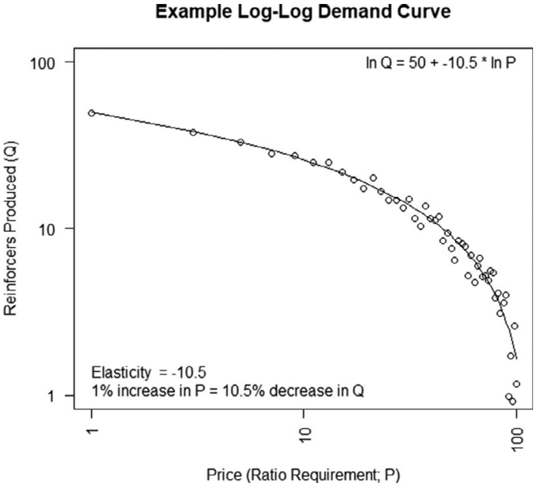
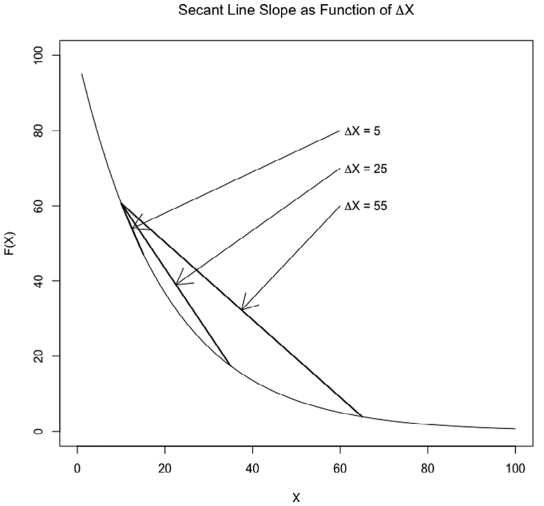


Figure 3
The Secant Line Approximations of Change in F(x) as a Function of Δx



Note. This estimate of the change in a function becomes increasingly exact as Δx approaches the limit.

that is, unit price. For convenience, an example of a linear demand function representing Q as a function of P is illustrated in Figure 2.

Derivative

In the most basic sense, the *derivative* of a function speaks to the rate of change in a function, for example, $f(x)$, a given point, that is, at x. Abstracting this to a demand function, the derivative speaks to the degree of change observed for a function, $f(x)$, per unit increase in x (Allen, 1938). This description is general because the derivative of a function can be expressed in several ways. In the most basic form, the derivative can be *approximated* via a secant line between two points along the curve (see Equation 1).

$$\frac{\Delta Y}{\Delta X} = \frac{f(X_2) - f(X_1)}{X_2 - X_1} \tag{1}$$

The ratio here shown above is a division of the degree of change in the function, ΔY, by the degree of change in x, that is, ΔX. As shown in Figure 3, as the value of ΔX approaches 0 the resulting slope converges to the *instantaneous* rate of change for the function (i.e., at x).

Although secant approximations provide a general estimate of the slope at a point (i.e., X_1), such estimates are not well suited to nonlinear functions (e.g., "S"-shaped curves) because these estimates inherently presume linear slope even when functions are nonlinear. Alternatively, the more appropriate approach in these cases is to solve for the instantaneous rate of change at a given point (i.e., the slope of tangent line). In this situation, the derivative is presented as follows in Equation 2:

$$\frac{df}{dx} = \frac{d}{dx} f(x) = f'(x) = y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{2}$$

Although analogous to the slope, it is necessary to explain the role of the limit in this method of the derivative. The limit here speaks to the lowest, most precise value of ΔX as h approaches zero. One cannot simply use zero here because division by zero is undefined. As an alternative to numerically estimating h using the terms here, terms may be differentiated such that h drops out of the solution. However, we note certain functions may not have a derivative while others could have many (i.e., different derivatives for different points). Regardless, if a limit exists for a function then that function can be differentiated and differentiation with respect to x for f

(x) yields the instantaneous rate of change for that function at x. This is the most commonly used approach because most functions can be differentiated. Prior to elaborating further, we note here that the notation of the derivative varies across fields and applications, with the Lagrange notation representing the (first) derivative as $f'(x)$ and the Leibniz notation as $\frac{df}{dx}$ or $\frac{d}{dx}f(x)$. As a matter of preference, the Leibniz notation will be used throughout this report.

Derivatives have thus far been reviewed as if the demand function took only a single factor (i.e., f was only as a function of x). Although studies of operant demand typically focus on demand as a function of P, economic research is rarely focused solely on P and demand is often modeled using several factors (e.g., price, income, and availability of substitutes). In these cases, differentiation performed with respect to a single parameter (e.g. x) alone would be considered a *partial* derivative because this would express rates of change as a function of that parameter, with all others held constant. The notation of the partial derivative differs from the base derivative and an example of this is shown below in Equation 3:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f\left(x, \alpha, Q_0, k\right) = f_x\left(x, \alpha, Q_0, k\right) \quad (3)$$

In addition to the examples provided here, further notation is provided in Appendix A.

Deriving Elasticity, η

The previous sections reviewed two relevant concepts, demand functions and derivatives. Clarification of these terms was necessary prior to discussing η because the (first) derivative of a function is not necessarily a reflection of η. That is, η speaks to *relative* changes between variables (e.g., Q and P) and there are multiple avenues for elucidating these relationships. In the interest of completeness, several common conventions for deriving η are discussed below.

Parameterized η

As briefly noted earlier, economists often evaluate demand using multiple linear regression and one can directly model η as a fitted parameter, assuming η is the same at any given P. In the simple model provided in Figure 2, namely, $\log(Q) = B_0 + B_1 \log(P)$, the fitted parameter β_1 is a direct representation of η because the dual logarithms of P and Q reflect *relative* changes (See Appendix B for a fully worked example). Should one solve for η in this case using derivatives, the solution ultimately reduces to β_1 and β_1 indicates that a 1% change in P is associated with a 10.5% decrease in Q. That is, in this case the parameter β_1 is a direct reflection of η. Regarding the specific fitting in Figure 2, the conclusion here would be that the demand for this good is highly (and singularly) elastic. Although a simple example is included here, these models are typically expanded to simultaneously evaluate η with respect to income, availability of alternative goods, and other factors that may influence consumption.

Log–Log Differentiation

In contrast to determining η via specific parameters, η is often determined through differentiation in the context of nonlinear models. In nonlinear models, the responsiveness between P and Q is not constant (i.e., not static) and η will vary as P changes. For instance, consider the Linear-Elasticity model demonstrated in Hursh et al. (1987). In this model, Hursh and colleagues presented a nonlinear model of demand as follows in Equation 4:

$$\log(Q) = \log(L) + b\log(P) - aP \quad (4)$$

In this model, L represents the predicted levels of consumption at a P of 1, b is the "initial downward slope of the demand curve" per Hursh et al. (1988), and a represents changes in slope as a function of P. In contrast with the parameterized approach, where η is a constant value, η here is not constant across increasing values of P. That is, η at a given P could be potentially inelastic ($\eta < |1|$), elastic ($\eta > |1|$), or unit elastic ($\eta = |1|$). This is distinct from the parameterized approach where a fixed value for η is represented as an individual constant parameter. Given that this model is nonlinear, it is logical no individual parameter represents η because η is not a single, fixed value that persists across prices. Ultimately, the process for deriving η here is via differentiation and the solution for η is as follows in

Equation 5 (see Appendix C for complete solution):

$$\eta = b - aP \tag{5}$$

The solution here requires differentiating P with respect to the logarithmic increases in price, that is, $\frac{d}{dP}\log(P) = \frac{1}{P}$. That is, η is not the first derivative for the Linear-Elasticity model and changes in P must be expressed as $\log(P)$. Rather, η here is determined using a partial derivative. When differentiated in log–log space, the instantaneous rate of change here reflects a unitless representation of η as a function of variables a, b, and P.

Although the Linear-Elasticity model has been used extensively in the literature, Hursh and Silberberg (2008) later presented the Exponential model of demand. The structure of the Exponential model is listed below in Equation 6:

$$\log(Q) = \log(Q_0) + k\left(e^{-\alpha Q_0 P} - 1\right) \tag{6}$$

Here, the rate constant α jointly reflects logarithmic changes in Q in conjunction with the intercept (Q_0) and the span parameter (k). In this more recent model, α alone indexes the rate of change whereas the earlier model jointly represents the rate of change with two parameters, a and b. Further, the incorporation of Q_0 in the exponent was included to support the standardization of P across reinforcers (Hursh & Silberberg, 2008). Despite these differences between models, differentiation here is also performed with respect to $\log(P)$ to evaluate *relative* changes in P and Q and the solution for η in the Exponential model is as follows in Equation 7 (see Appendix D for complete solution):

$$\eta = -aQ_0 kPe^{-aQ_0 P} \tag{7}$$

Linear–Linear Differentiation

Although each of the preceding methods for deriving η has used logarithms to evaluate relative changes in P and Q, η can also be derived using the linear (natural) scale (e.g., Koffarnus et al., 2015; Yu et al., 2014). However, η speaks to relative changes between variables and values of both P and Q in the linear scale must be adjusted such that changes in prices and consumption are relative (i.e., not absolute). That is, the absolute changes expressed on the linear scale can be transformed to reflect percentage changes. For instance, consider the Exponentiated model proposed by Koffarnus et al. (2015). Briefly, this model is a restatement of the Exponential model proposed by Hursh and Silberberg (2008) with model terms (i.e., Q_0) exponentiated to the linear scale. The structure of this model is noted below in Equation 8:

$$Q = Q_0 * 10^{k*\left(e^{-\alpha Q_0 P} - 1\right)} \tag{8}$$

The incorporation of Q on the linear scale has the benefit of accommodating zero consumption values, however, evaluating Q on the linear scale requires additional steps to ensure differences are relative. That is, the derivative of this demand function with respect to P reflects responsiveness in terms of *absolute* changes, or η'. The determination of η' is indicated below in Equation 9:

$$\eta' = \frac{\Delta Q}{\Delta P} = \frac{\partial}{\partial P} f\left(Q_0, k, \alpha, P\right) \tag{9}$$

The difference between η' and η is that η is unitless and η' is not. Fortunately, units here can be negated by multiplying the absolute responsiveness (η') by the respective P by the predicted level of Q at given P. This is illustrated below in Equation 10 (see Appendix E for a complete solution).

$$\eta = \eta' \frac{P}{Q} \tag{10}$$

In performing these operations, the absolute changes in P and Q are instead reflected as percentage change, and thus, a relative and unitless representation of responsiveness between two variables. In working through this example without logarithms, it warrants reiterating that multiple methods are available for deriving η but solutions are ultimately specific to the scale and units used in each instance. Further, it warrants noting that the exact solution for unit elasticity proposed in Gilroy et al. (2019) is robust to scale and unit differences and applies equally to both the Exponential and Exponentiated models.[2]

Clarifying η in Operant Demand

The preceding sections served to illustrate η and how η retains a consistent derivation regardless of model structure, theoretical perspective, or specific variables. Despite differentiation serving as the basis for η in demand functions, the term "demand elasticity" has emerged in studies of operant demand as a *general* reference to the steepness or rapidness of change in Q as a function of P. Although both η and "demand elasticity" each speak to a responsiveness of changes in Q to changes in P, it warrants reiterating η has a specific mathematical basis while references to "demand elasticity" have been used in the context of relatively ranking "steepness" or rates of change (e.g., high vs. low α). To make this comparison clearer, we direct the reader to Hursh and Silberberg (2008) where the authors state "What is needed is a new equation that maintains the predictive successes of the linear-elasticity equation but addresses the need of having a single parameter defining changes in elasticity of demand" (p. 190). Here, we read and infer that the original intent of Hursh and Silberberg was to derive a singular parameter not to reflect η directly but to reflect *changes* in η. That is, higher values of α represent more rapid changes in Q while lower values would represent more gradual changes. However, absent clarification between these, we have seen repeated instances in the literature wherein authors seemingly regard α as synonymous with η.

Revisiting the mathematical basis for η, it is clear α alone cannot represent η because α is a fixed value across prices while η is dynamic. Revisiting Hursh and Silberberg (2008) again, they stated, "The slope of the demand curve, elasticity, is jointly determined by k and α, but because k is a constant, changes in elasticity are determined by the rate constant, α" (p. 191). Here, the authors describe α as a value that indexes change in η, while η is ultimately a product of various terms (see Appendix D for a worked solution). In revisiting this statement, early accounts of α do not explicitly state that α and η are distinct measures. Barring a more complete and explicit description of η, some have presumed α in this model represents η and this is not true.

Clarifying Parameter α

As noted earlier, α and η are often communicated interchangeably when referring to "demand elasticity" in studies of operant demand. Newman and Ferrario (2020) visited this issue as well, noting "…elasticity is a well-defined and useful concept in economics, but α is not a measure of elasticity. Rather, α is a measure of the price at which the animal performs maximum work, equivalent to the quantity P_{MAX}" (p. 949). Referring to the description provided in Hursh and Silberberg (2008), parameter α was originally described as a measure representing "changes in elasticity." The function of α is more accurately described as representing changes in P_{MAX} and this relationship is made clearer when reviewing the solution provided in Gilroy et al. (2019). This solution, shown in Equation 11, solves for P_{MAX} ($\eta = -1$) using the parameters derived from either the Exponential or Exponentiated model of operant demand.

$$P_{MAX} = \frac{-W_0\left(-\frac{1}{\log 10^k}\right)}{aQ_0} \tag{11}$$

Shown here, the omega function (W_0) is applied to a transformation of the span parameter k and this value is divided by the product of a and Q_0 to determine P_{MAX}. From here, it is simple to rearrange the terms such that a similar solution is possible for a as well:

$$a = \frac{-W_0\left(-\frac{1}{\log 10^k}\right)}{Q_0 P_{MAX}} \tag{12}$$

The solutions here highlight how, holding k and Q_0 constant, both P_{MAX} and α are perfectly and inversely rank ordered with one another (i.e., larger α, smaller P_{MAX}). That is, those factors held constant, P_{MAX} and the inverse of α, $\frac{1}{\alpha}$, will maintain a perfect rank

[2] We note that the exact solution to P_{MAX} is amenable to both Exponential and Exponentiated models but the two will naturally provide different estimates when applied to different sets of data (e.g., with and without zero values).

order relationship with one another.[3] Hursh and Roma (2013) highlighted this relationship and noted that this trait of α served as a form of EV and supported an early approximate of P_{MAX} in the Exponential model of operant demand. The original approximation was possible because α speaks to how rapidly the demand curve approaches the point of maximum responding, P_{MAX}. Given the mathematical link between α and unit elasticity, it is more appropriate to note that the changes captured in α are more accurately described as reflecting changes in P_{MAX} (or maximum responding) rather than changes in η more broadly.

Clarifying P_{MAX} and Unit Elasticity

When we speak of P_{MAX}, this measure reflects the P value at which an organism responds at the highest rates to produce the reinforcer (i.e., exerts most work). Generally speaking, η at P_{MAX} is typically −1 in the Exponential and Exponentiated models of demand, but instances exist wherein $P = P_{MAX}$ but $\eta \neq -1$. As discussed in Gilroy et al. (2019), no real solutions exist for $\eta = -1$ in situations where the span parameter k value is below 1.18, i.e. $\frac{e}{\log(10)}$. Were the units of k the natural log rather than $\log_{10}$, the lower limit would be e. This limit is logical given that the demand curve must decrease at least 1 log unit decrease from Q_0 in response to 1 log unit increase of P to produce an η of −1. That is, apart from variability in how researchers prepare this parameter (Kaplan, Foster et al., 2018), the span constant also has an unintended effect of limiting η (Newman & Ferrario, 2020).

In such cases where k is defined below the limits noted above, a P_{MAX} indeed exists (i.e., a price where maximum responding is observed) but η at this point *cannot* be −1. Consider the following example: A demand series is fitted to the Exponential model of operant demand with three separate k values: 1, 1.5, and 2. Per the solution from Gilroy et al. (2019), a solution for η of −1 exists only for the k values of 1.5 and 2. Newman and Ferrario (2020) also noted this limitation and provided a mathematical basis for the absolute lower limit of η given k, η_{MAX}, and their solution is provided below in Equation 13[4]:

$$\eta_{MAX} = -\frac{k}{e} \tag{13}$$

Returning to our example of demand curves fitted with varying span constants, the η_{MAX} for each is displayed in respective plots in Figure 4. Here, we see k sets the lower limit on the range of η possible across prices. Given this limitation, the value of the span constant may unintentionally introduce a situation where researchers are unaware that it is mathematically impossible for η to equal −1. In this situation, η at the observed P_{MAX} in this situation would most likely be at or near the η_{MAX}, given the span constant because no real solution exists for $\eta = -1$. In such a situation, parameter α will continue to speak to a point of maximum responding but the link between P_{MAX} and $\eta = -1$ will be lost.

Future Directions in Operant Demand

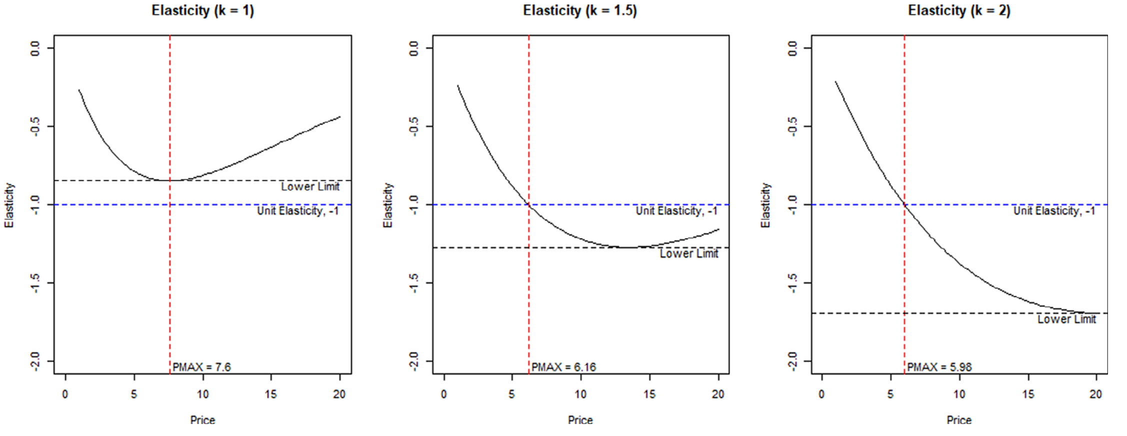
The operant demand framework has enhanced the ability of researchers to evaluate human and nonhuman responding under a variety of constraints (e.g., prices, substitutes available). This framework has rapidly grown to include a variety of experimental and hypothetical purchase measures (Bickel et al., 2018; Kaplan, Foster et al., 2018), but several aspects of this emerging methodology warrant further refinement and clarification as this framework continues to expand. Principal among areas to clarify, η in operant demand has been communicated in various ways and this detracts from a consistent interpretation of research findings across labs and across domains. That is, although loose references to "demand elasticity" may not alter scientific conclusions within individual experiments, imprecise references may lead to miscommunication of η across studies and disciplines. For example, systematic meta-analyses of "demand elasticity" could theoretically be summaries of EV, α, or P_{MAX} and potentially never summarize η. Speaking of all these metrics interchangeably inevitably

[3]We note that parameter k is likely to remain constant across fitted curves in an experiment; P_{MAX} is likely to covary with parameter Q_0 due to differences associated with varying dosages.

[4]We note here that the lower limit proposed by Newman and Ferrario (2020) put k in base units of e. The $\log_{10}$ equivalent would simply replace e with $\frac{e}{\log(10)}$.

Figure 4

A Demand Function with the Same Data Evaluated Using Varying Span Constants



Note. As displayed here, k affects the maximum lower limit of η and this determines whether or not unit elasticity can be reflected in the demand curve. [Color figure can be viewed atwileyonlinelibrary.com]

frustrates a true synthesis of how η of demand for reinforcers varies within and across various disorders (e.g., alcohol abuse, illegal drug use). Apart from limiting research synthesis in the behavioral sciences, loose references to “demand elasticity” also limits the ability of researchers to clearly communicate with other fields where η has a clear and precise interpretation (e.g., economics). For these reasons, we recommend that researchers adopt a common, more consistent definition of these parameters. Regarding α, we have found it more accurate to refer to this as an index of the *rate of change* in η, given the span of the demand curve (k) and the base level of demand intensity (Q_0). This definition clearly articulates how α *relates* to η as a function of other parameters (i.e., it is inversely related to P_{MAX}). Similarly, we have found it more appropriate to present P_{MAX} as the predicted or observed P that reflects peak levels of responding (i.e., maximum output). This definition is superior to describing P_{MAX} as unit elasticity because P_{MAX} is an explicit value of P and because η is restricted in cases where constant k exists below the recommended lower limits (i.e., not always unit elastic). Lastly, we believe that η is most clearly defined as the *responsiveness* of changes in Q to changes in P. Although general, a broad definition is warranted because η ultimately varies across P in operant demand and because *unit elasticity* is only one instance of η.

In addition to clarifying aspects of demand curve analyses, this report further elaborates upon the numerous challenges associated with an explicit span parameter in demand curve analyses. Namely, issues associated with k and η present a continued and historical source of variability in operant demand. Furthermore, it is unknown to what degree misspecified k values (i.e., below recommended lower limit) have influenced subsequent syntheses of behavioral economic works. Looking forward, the issues with an explicit span parameter naturally prompt a reevaluation of whether an explicit span parameter supports a consistent and replicable approach to understanding response–reinforcer relationships. That is, it may be necessary to pursue alternative analytical strategies free from this parameter.

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Appendix A

Notation of Derivative

$$\frac{df}{dx} = \frac{d}{dx}f(x) = f'(x) = y' = \lim_{h \to 0}\frac{f(x+h) - f(x)}{h}$$

Notation of Partial Derivative

$$\frac{\partial f(x, \alpha, Q_0, k)}{\partial x} = \frac{\partial}{\partial x}f(x, \alpha, Q_0, k) = f_x(x, \alpha, Q_0, k)$$

Appendix B

Deriving η from Individual Parameters

$$\log(Q) = \beta_0 + \beta_1 \log(P)$$
$$\frac{d}{dP}\beta_0 + \beta_1 \log(P) = \frac{\beta_1}{P}$$
$$\frac{d}{dP}\log(P) = \frac{1}{P}$$
$$\eta = \frac{\frac{d}{dP}\beta_0 + \beta_1 \log(P)}{\frac{d}{dP}\log(P)} = \frac{\frac{\beta_1}{P}}{\frac{1}{P}} = \frac{\beta_1}{P}\frac{P}{1} = \frac{\beta_1 P}{P} = \beta_1$$

Appendix C

Deriving η in the Linear Elasticity model of Demand

$$\frac{\partial}{\partial P}f(L, a, b, P) = \frac{b}{P} - a$$
$$\frac{d}{dP}\log(P) = \frac{1}{P}$$
$$\eta = \frac{\frac{\partial}{\partial P}f(L, a, b, P)}{\frac{d}{dP}\log(P)} = \frac{\frac{b}{P} - a}{\frac{1}{P}} = \frac{\frac{b}{P} - a}{1}\frac{P}{1} = b - aP$$

Appendix D

Deriving η in the Exponential model of Demand

$$\frac{\partial}{\partial P}f(Q_0, k, \alpha, P) = -aQ_0 k e^{-\alpha Q_0 P}$$
$$\frac{d}{dP}\log(P) = \frac{1}{P}$$
$$\eta = \frac{\frac{\partial}{\partial P}f(Q_0, k, \alpha, P)}{\frac{d}{dP}\log(P)}$$
$$= \frac{-aQ_0 k e^{-aQ_0 P}}{\frac{1}{P}} = -aQ_0 k e^{-aQ_0 P}\frac{P}{1}$$
$$= -aQ_0 k P e^{-aQ_0 P}$$

Appendix E

Casting Absolute Changes in terms of Unitless Change

$$\eta' = \frac{\Delta Q}{\Delta P} = \frac{\partial}{\partial P}f(Q_0, k, \alpha, P)$$
$$\eta = \frac{\%\Delta Q}{\%\Delta P} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q \frac{1}{Q}}{\Delta P \frac{1}{P}}$$
$$= \frac{\Delta Q}{\Delta P}\frac{P}{Q} = \frac{\partial}{\partial P}f(Q_0, k, \alpha, P)\frac{P}{Q} = \eta'\frac{P}{Q}$$

Deriving η in the Exponentiated model of Demand

$$\frac{\partial}{\partial P}f(Q_0, k, \alpha, P)$$
$$= -10k\left(e^{-\alpha Q_0 P} - 1\right)\alpha k Q_0^2 e^{-\alpha Q_0 P}\log(10)$$
$$\frac{d}{dP}P = 1$$
$$\eta = \frac{\frac{\partial Q}{Q}}{\frac{\partial P}{P}} = \frac{\frac{\partial}{\partial P}f(Q_0, k, \alpha, P)}{\frac{d}{dP}P}\frac{P}{Q}$$
$$= \frac{-10k(e^{-\alpha Q_0 P} - 1)\alpha k Q_0^2 e^{-\alpha Q_0 P}\log(10)}{1}\frac{P}{Q}$$