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An Exact Solution for Unit Elasticity in the Exponential Model of Operant Demand

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Research applying the behavioral economic demand framework is increasingly conducted across disciplines. With respect to psychopharmacology and substance abuse, real and hypothetical purchase tasks are regularly used to evaluate the demand for various substances and reinforcers, such as alcohol. At present, a variety of methods has been introduced to solve for the point of unit elasticity, or P_{max}, in the exponential model of demand; however, these methods vary in their potential for error. Current methods for determining P_{max} are presented here and a novel exact solution for P_{max} in the exponential model of demand is introduced. This solution provides an exact calculation of P_{max} using the omega function, as algebraic solutions are not possible. This novel approach is introduced, discussed, and systematically compared to earlier methods for determining P_{max} using computer simulations and reanalyses of published study data. Systematic comparison indicated that this new approach, an exact analytic solution for P_{max}, provides results that are identical to computationally intensive P_{max} methods that directly evaluate the slope of the demand function. The exact analytic P_{max} approach is reviewed, its calculations explained, and an easy-to-use web tool is provided to assist researchers in easily performing this calculation of P_{max}. Implications for reducing potential sources of error are reviewed and future directions are also discussed.

Public Health Significance
This study proposes and explains an improved method for calculating unit elasticity of demand. While approximate methods exist, even small sources of error due to estimation have negative implications for the development of public policy. Using this new method, an exact calculation of unit elasticity can now be obtained much more easily and reliably.

Keywords: behavioral economics, reinforcer efficacy, exact solution, computer software, simulation

Classical economic theory employs demand analyses to understand market influence on consumers' willingness to pay for particular goods and services. Central to demand theory is the notion of demand elasticity, which is defined as the "ratio of the

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relative change in a dependent variable to the relative change in an independent variable" (Watson & Holman, 1977, p. 34). Behavioral economists have translated this concept to the consideration of reinforcement operations on operant responding. Operant behavioral economics, namely the methods specific to studying operant demand, provides a framework for quantifying response-reinforcer relationships under some type of constraint (number of responses required per unit of reinforcer, delay to reinforcement, effort associated with responding for one unit of reinforcement, etc., e.g., Hursh, 1980, 1984; Kagel & Winkler, 1972; Rachlin, Green, Kagel, & Battalio, 1976). Demand elasticity in operant psychology may thereby be used to provide a quantification of a reinforcer's hedonic or motivational value.

The relationship between the consumption of reinforcers (e.g., alcohol, nicotine) and the responding necessary to produce them is complex, though it has been effectively modeled in several ways (Hursh, Raslear, Bauman, & Black, 1989; Hursh & Silberberg, 2008; Koffarnus, Franck, Stein, & Bickel, 2015). Regardless of the specific model used to quantify the demand for reinforcers, the operant demand approach (hereafter considered synonymous with the simple term *demand*) has been particularly useful in the context of various willingness-to-pay tasks—particularly purchase tasks (Roma, Reed, DiGennaro Reed, & Hursh, 2017). Using these tasks, the demand for substances or goods such as alcohol (Kaplan, Foster, et al., 2018), nicotine (MacKillop & Tidey, 2011; Stein, Koffarnus, Stepanov, Hatsukami, & Bickel, 2018), prescription drugs (Pickover, Messina, Correia, Garza, & Murphy, 2016), or marijuana (Aston, Metrik, & MacKillop, 2015; Collins, Vincent, Yu, Liu, & Epstein, 2014) is assessed at various financial costs and inferences are drawn based on the degree to which participants will defend their levels of consumption as prices increase (Hursh, 1980, 1984).

While the operant demand framework has been particularly useful in characterizing abuse liability for drugs and other addictive substances (e.g., nicotine, alcohol), this methodology has also been extended to other areas of choice and decision-making. For example, these methods have also been used to evaluate food intake and dietary choices (Epstein et al., 2018; Epstein & Saelens, 2000; Epstein, Smith, Vara, & Rodefer, 1991; Saelens & Epstein, 1996), use of indoor tanning services (Reed, Kaplan, Becirevic, Roma, & Hursh, 2016), and purchasing groceries (Foxall, Olivera-Castro, Schrezenmaier, & James, 2007; Foxall, Wells, Chang, & Oliveira-Castro, 2010). Further, other researchers have explored areas such as workforce attrition and incentives (Henley, DiGennaro Reed, Kaplan, & Reed, 2016; Henley, DiGennaro Reed, Reed, & Kaplan, 2016), "green" consumerism (Kaplan, Gelino, & Reed, 2018), and informing interventions for individuals with disabilities (Gilroy, Kaplan, & Leader, 2018; Reed, Kaplan, & Becirevic, 2015; Reed et al., 2009) using this approach. On a macrolevel, this framework has been used at the population level, providing a means of developing and evaluating empirically supported public policy (Guthrie, 2017; Hursh, 1991; Hursh & Roma, 2013; MacKillop et al., 2012; Reed et al., 2016).

Demand Curve Analyses

Current methods for quantifying the strength, or potency, of a reinforcer (e.g., alcohol, nicotine) represent this quality as a curve, whereby the overall consumption of a reinforcer (Q) slopes downward as a nonlinear function of increasing cost (P; Hursh, 1980, 1984). Prior to representing reinforcer strength in this way, earlier methods compared the potency of reinforcers based on measures of *relative* efficacy (Johnson & Bickel, 2006). For example, the efficacy of a reinforcer such as nicotine (e.g., cigarettes) might be compared to some alternative reinforcer (e.g., money) based on some aspect of responding under constraint, which might include peak levels of responding (i.e., highest levels of expenditure), reinforcer breakpoint (i.e., highest costs endured), or some other pattern of responding, such as preference (Katz, 1990). These earlier methods, which compare reinforcers based on one aspect of the response–reinforcer relationship, have since been superseded by demand curves, which represent reinforcer efficacy as a multidimensional construct with metrics arising from a fitted curve (Bickel & Madden, 1999; Bickel, Marsch, & Carroll, 2000; Johnson & Bickel, 2006).

Representing reinforcer strength as a curve offers several advantages over measures of relative reinforcer efficacy (Johnson & Bickel, 2006; Reed, Niileksela, & Kaplan, 2013). For example, modeling changes in consumption as a function of changes in price as a curvilinear function serves to integrate various aspects of the response-reinforcer relationship in a single, unified approach. Further, modeling the demand in this way reveals additional qualities of reinforcers. Among these, the demand curve permits an analysis of the elasticity of demand for a reinforcer (Hursh, 2014; Hursh & Silberberg, 2008; Lea, 1978; Lea & Roper, 1977). Briefly, the demand for reinforcers is differentially influenced by constraints such as price and time. Changes in consumption as a function of changes in the cost–benefit ratio are referred to as the elasticity of demand and index the degree to which consumption is sensitive to these increasing costs. That is, there are regions of the demand curve where consumption is relatively unaffected by increases in costs and others where consumption is substantially affected by increases in cost (see Figure 1). These regions are termed the *inelastic* and *elastic* ranges of the demand curve, respectively, and prices associated with each range exert differential effects on consumption. Demarcating these two regions of the demand curve is the point of *unit elasticity* or $P_{\max}$, a location upon the demand curve whereby one log-unit increase in price is accompanied by one log-unit decrease in levels of predicted consumption (i.e., -1 unit consumption/1 unit price $= -1$ unit change). This ratio (i.e., slope) is near zero at low prices and grows increasingly negative as larger changes in consumption take place. An example of this slope and the calculations involved are illustrated in the right panel of Figure 1. In this example, unit prices less than $P_{\max}$ are associated with smaller changes in levels of consumption when prices increase, that is, $-1 < f'(x) < 0$, and costs greater than $P_{\max}$ are associated with larger changes in consumption when prices increase, that is, $f'(x) < -1$.[1]

Approximations of Unit Elasticity

The exponential (Hursh & Silberberg, 2008) and exponentiated (Koffarnus et al., 2015) models are frequently used in studies of

[1] We make note that $f'(x)$, or f-prime, here refers to the first order derivative of the exponential demand function and that calculations based on the first order derivative have been considered "exact" methods for determining $P_{\max}$.

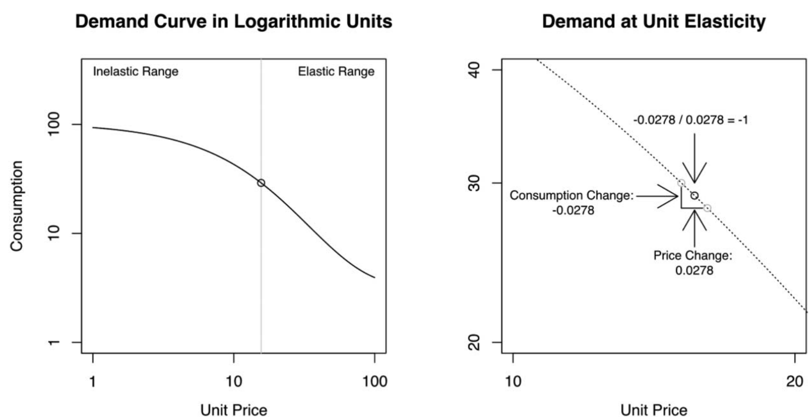

Figure 1. Demand curve and P_{max} in log–log space. These figures illustrate the calculation of unit elasticity (P_{max}) in log–log space. The left panel illustrates the form of a conventional demand curve and the right panel illustrates the specific calculations involved. As described in the right panel, a slope of −1 on the demand curve indicates that 1 log-unit increase in price equates to a −1 log-unit decrease in levels of consumption.

operant demand.[2] In both models, the shared methods for determining P_{max} have varied in several ways. Before discussing these ways, we make note that calculations of model slope are ultimately the same for both the exponential and exponentiated models. This is because elasticity is assessed in log–log space and scaling the exponentiated model of demand into log–log space ultimately provides the same projection. As such, we reference the exponential model directly in this work because this model already occupies the necessary space for calculating model slope (i.e., log–log) by default. However, we reiterate that regardless of whether the exponential or exponentiated model is used, P_{max} calculations would be performed in the same manner. The exponential model takes the following form:

$$\log_{10}Q = \log_{10}Q_0 + k(e^{-\alpha \times Q_0 \times x} - 1)$$

As first noted by the exponential model authors (Hursh & Roma, 2013; Hursh & Silberberg, 2008), an arithmetic solution for model slope (i.e., P_{max}) is not available. The terms of this model's first order derivative prohibit an arithmetical solution for the price (i.e., x) at a slope of −1 because the term for price (i.e., x) appears twice in the derivative—both inside and outside of the exponent. As a workaround for this issue, alternative approaches for calculating (or approximating) P_{max} have been introduced. First, P_{max} can be approximated empirically by "observing" a point of maximum responding (i.e., the empirical O_{max}). In this approach, the empirical O_{max} is calculated by multiplying the levels of recorded consumption by the corresponding prices. From the resulting values, the highest is termed the *empirical* O_{max} and this measure can be used to approximate an "observed" P_{max} for an empirical demand curve, because O_{max} and P_{max} are related (Greenwald & Hursh, 2006). This approach is driven by the available data alone, separate from models of demand, and provides an indicator of unit elasticity by identifying the price associated with the highest levels of responding. Second, there are mathematical formulas that result in an approximation of P_{max} using the fitted model parameters (Hursh, 2014; Hursh & Roma, 2013). The approximate P_{max} formulas produce values that are highly correlated with the "true" or exact P_{max}, which would be found if directly evaluating model slope using specialized programming. Lastly, an exact P_{max} can be determined using specialized software wherein a program systematically searches for the price where the first order derivative is equal to −1 (Hursh & Roma, 2013).[3] In this approach, referred to here as derivative P_{max}, an algorithm is used to iteratively search for P_{max} using a fitted demand function. While several methods are available for determining P_{max}, there are inevitable variations in the values resulting from each of these methods and this variability is inherently due to difficulties solving for an exact model slope of −1. Any variations in P_{max} values present challenges for comparing values across studies and for the accuracy of making population-level recommendations (e.g., formulating public policy related to substance abuse).

An Exact Solution for Unit Elasticity

While there is no arithmetic solution for P_{max}, there are alternatives where a slope of −1 can be determined without the need for specialized computer programs. For example, the Lambert *W* (i.e., omega) function can be used with the first order derivative of the exponential model of demand to solve for P_{max}. Briefly, the *W* function allows for the solving of *x* when a function takes the form of $y = xe^x$. This is desirable in the case of the first order derivative of the exponential model of demand because this applies to the *x* terms that exist inside and outside of the exponent. Using the W function, this simple example then takes the form of $x = W(y)$. As such, the same logic may be applied to the first order derivative for P_{max}.

The purpose of this study was to evaluate the accuracy, reliability, and utility of an exact solution for P_{max} in the exponential model of demand—hereafter referred to as *analytic* P_{max}. To evaluate this novel approach, computer simulations and published

[2] While the exponential and exponentiated models are presented separately, it warrants noting that the two models share a common basis but differ in scale. That is, the exponential model exists in log scale and the exponentiated model is in linear scale.

[3] We note that derivative-based approaches for determining P_{max} have also been referred to as the exact P_{max}.

data were used to facilitate comparisons of the various methods for calculating P_{max}. Specifically, this study asked the following questions: (a) To what degree do the observed, approximate, derivative, and analytic methods of calculating P_{max} relate to one another? (b) To what degree does the analytic method of calculating P_{max} provide results consistent with the derivative, or exact, P_{max} from which it was derived? (c) To what degree do results from the P_{max} methods correlate with other behavioral indicators of abuse liability in published study data?

Method

Simulated Data Series

A total of 1,000 simulated consumption series were generated from the results of an earlier peer-reviewed study on decision-making (Kaplan & Reed, 2018) and all simulations were conducted using the R statistical program (R Core Team, 2017). Participants in Kaplan and Reed (2018) were recruited using the Amazon Mechanical Turk platform to complete a hypothetical alcohol purchase task (APT; Kaplan, Foster, et al., 2018). Although the purpose of the source study was to investigate the influence of "happy hour" specials on self-reported alcohol purchases, only data from the standard APT were used as a basis for computer simulation. The following prices were included in the APT: $0.00 (free), $0.25, $0.50, $1.00, $1.50, $2.00, $2.50, $3.00, $4.00, $5.00, $6.00, $7.00, $8.00, $9.00, $10.00, $15.00, and $20.00. Simulated consumption at each price point was constructed using means and standard deviations in overall responding at that respective price point. Simulated consumption series that met criteria for systematic responding (Stein, Koffarnus, Snider, Quisenberry, & Bickel, 2015) and R^2 values greater than 0.8 were included in the simulations used to compare unit elasticity methods. A complete description of the computer simulation, as well as the source code necessary to reproduce the data and analyses, is provided in the GitHub repositories of Shawn P. Gilroy.

Peer-Reviewed Study Data

Data from a peer-reviewed study were used to evaluate relationships between individual calculations of P_{max} and related behavioral indicators of substance abuse. Using data from Kaplan and Reed (2018), the elasticity of demand was determined from responding on the APT and the results from each method of calculating P_{max} were compared with related indicators of alcohol consumption. Specifically, Kaplan and Reed (2018) administered the Daily Drinking Questionnaire (Collins, Parks, & Marlatt, 1985), which assessed self-reported weekly instances binge-drinking, number of drinks consumed, and total number of hours spent drinking. Although Kaplan and Reed (2018) administered multiple APTs (e.g., with and without drink specials), only the initial, standard version was used in the present study. The prices included in these data were identical to those used in the simulated data series and a total of 1,104 subjects participated in the study.

Nonlinear Model Fitting

Demand curve analyses were performed using the *beezdemand* R package (Kaplan, Gilroy, Reed, Koffarnus, & Hursh, 2018), a peer-reviewed tool that features various modeling methods specific to operant demand. Individual Q_0 and α values were fitted using *FitCurves* at default settings for the exponential model of demand. Optimization in *beezdemand* was performed using the Gauss–Newton optimizer included in the default nonlinear curve-fitting method (*nls*) in the R program (R Core Team, 2017). Scaling constant k was determined separately for each simulated series by subtracting the minimum level of consumption from the maximum level of consumption, each in $\log_{10}$ units, and adding a value of 0.5. A value of 0.5 was added to this range to minimize risks associated with using a k value that does not reflect the full range of observed consumption values (Gentile, Librizzi, & Martinetti, 2012; Kaplan, Foster, et al., 2018).

Calculations of Unit Elasticity

Four methods for determining P_{max} were evaluated and all calculations were performed using the R statistical program (R Core Team, 2017). All materials used to simulate participants, calculate P_{max}, and perform analyses have been open sourced, and details for acquiring these resources are provided in Shawn P. Gilroy's GitHub repositories. Each method included in this evaluation is listed and explained below.

Observed P_{max}. As an alternative to evaluating the slope of a fitted model, or numerically approximating it, an "observed" P_{max} can be inferred from the highest levels of observed responding, the empirical O_{max} (Greenwald & Hursh, 2006). As P_{max} is related to O_{max}, the empirical O_{max} (i.e., based solely on observed data) can be assumed to represent the ordinate of unit elasticity while P_{max} would be represented by the abscissa (i.e., price). In this way, the observed P_{max} infers model slope without a model by using the location of the empirical O_{max} on the x-axis (i.e., price). Numerically, the total levels of responding are calculated at each measured price point by multiplying levels of consumption by the corresponding unit price. This provides a series of values that reflect the maximum levels of responding or work output at each price point. From these values, the unit price associated with the highest levels of responding, or maximum work output, is considered to represent the observed P_{max}. Additionally, we note that in cases where more than one O_{max} was observed, the lower unit price was considered the observed P_{max}, consistent with earlier peer-reviewed software (Kaplan, Gilroy, et al., 2018).

Approximate P_{max}. First introduced along with the exponential model demand (Hursh & Silberberg, 2008), and later revised (Hursh, 2014), the unit price where a demand curve reaches unit elasticity can be approximated numerically using fitted model parameters.[4] This calculation is not derived from the slope, per se, though it results in a value that closely approximates the true P_{max}. The latest form of approximate P_{max} is calculated as follows:

$$P_{MAX} = \frac{1}{Q_0 \times \alpha \times k^{1.5}} \times (0.083 \times k + 0.65)$$

Limitations of this approach have been noted by model authors, namely, that error varies significantly with respect to the size of scaling parameter k, and thus, the range observed in overall levels

[4] In addition to using fitted model parameters, the approximate P_{max} uses additional constants that were determined by the original authors to improve slope approximation.

of consumption. However, despite limitations, this calculation has been found to be a good approximation for many combinations of fitted demand parameters (Hursh, 2014; Hursh & Roma, 2013).

Derivative P_{max}. In contrast to numerical approximates, the slope of the exponential demand curve can be iteratively evaluated using its first order derivative and specialized computer programming. The results of the approach have been previously referred to as a true or exact P_{max} (Hursh & Roma, 2013), as results are determined using a computationally exhaustive process that directly evaluates model slope at various prices until the first order derivative equals −1. The first order derivative, as provided by Hursh and Silberberg (2008), takes the following form:

$$f'(x) = ln10^k \times (-\alpha \times Q_0 \times x \times e^{-\alpha \times Q_0 \times x})$$

Although not required, this method can be adapted into a more easily optimized loss function by adding a constant of 1 and taking the absolute value of the result. A visual comparison of these two objective functions is provided in Figure 2. In effect, this modification represents P_{max} as a zero value when the slope of the demand function is −1 (i.e., −1 + 1 = 0). Further, taking the absolute value produces a *V*-shaped function wherein the lowest point in this function represents P_{max}. This form improves the speed and simplicity of an optimization routine, which iteratively searches for the price wherein the loss function is at its minimum. This loss function was used along with the default minimization method in R, *optim*, using a port of the Broyden–Fletcher–Goldfarb–Shanno algorithm (Nash, 1990).

Analytic P_{max}. As an alternative to empirical, approximate, and iterative computer methods, P_{max} can be calculated analytically using the Lambert *W* function (also known as the omega function). Simply put, the terms of the first order derivative provided by (Hursh & Roma, 2013) can be rearranged (where α, Q_O, and constant *k* are known) so that unit price can be solved at a slope value of −1. The *W* function can be used to address the *x* term appearing inside and outside of the exponent, which was a barrier reported previously. Using the *W* function, one can construct the form required to use the *W* function (i.e., $y = xe^x$) by rearranging several terms, as follows:

$$y = ln10^k \times (-\alpha \times Q_0 \times x \times e^{-\alpha \times Q_0 \times x})$$
$$-1 = ln10^k \times (-\alpha \times Q_0 \times x \times e^{-\alpha \times Q_0 \times x})$$
$$\frac{-1}{ln10^k} = -\alpha \times Q_0 \times x \times e^{-\alpha \times Q_0 \times x}$$

In these equations, the necessary form to use with the *W* function can be prepared so that a solution for a slope of −1 is possible. The final solution for P_{max} using the *W* function is as follows:

$$P_{MAX} = \frac{-W_0(-1/ln10^k)}{\alpha \times Q_0}$$

While well-suited to this application, it warrants noting that the Lambert *W* function is complex and multiple branches and solutions can exist (i.e., real and imaginary). However, for our purposes, we will use the primary branch of this function, as denoted by W_O. This branch has both real and imaginary solutions and this method of solving for P_{max} is possible so long as the value used in W_O exists within the following range:

$$-e^{-1} < \frac{-1}{ln10^k} < 0$$

Put simply, this approach results in an exact calculation of P_{max} provided that the *k* used to fit the model exists above a certain lower limit. In this approach, especially small *k*s would push the value supplied to W_0 outside of the range specified above and into a region where no real solutions exist. Solving for this absolute lower limit, an analytic P_{max} can be calculated in all cases where constant *k* that exists above a lower limit of 1.180535 and the exact determination of this value is provided below:

$$\frac{e}{ln10} < k < \infty$$
$$1.180535 < k < \infty$$

Given that this novel approach is an exact solution for the derivative approach, the analytic method should provide results that are identical to the derivative method without the need for specialized computer programming. In contrast, analytic P_{max} may be performed using scientific calculators or customized spread-

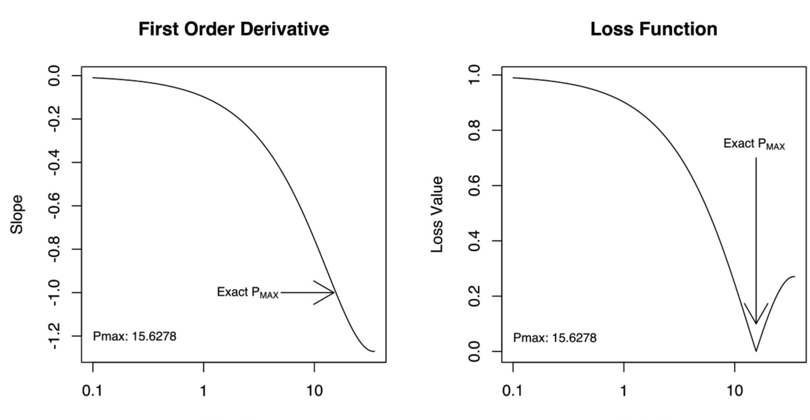

Figure 2. Model slope and modified loss function. This figure illustrates a method of solving for P_{max} using the first derivative. The left panel illustrates the value of the first derivative (i.e., slope) and the right panel illustrates a modified equation (i.e., a loss function) that represents a slope value of −1 as a zero point, which can then be minimized to determine exact P_{max} at a value of 0.

sheet software. The specific methods used to perform the W function in this study were derived from the GNU Scientific Library, an open-source library of mathematical methods (Gough, 2009).[5]

Data Analysis Plan

The methods described here were systematically compared to evaluate the accuracy, reliability, and correspondence between calculations of unit elasticity. Each of the individual unit elasticity calculations described above was performed for each of the 1,000 simulated series and correspondence was assessed using Pearson correlations and scatterplots. Scatterplot comparisons were constructed to illustrate the correspondence between simulated measures and correlations were calculated overall as well as with select, well-fitting models ($R^2 > .9$). Additionally, each method was applied to data from a published, peer-reviewed study to compare how each measure of unit elasticity related to various participant factors. Relationships between P_{max} methods and participant factors were assessed using Spearman correlations.

Results

The results of simulated comparisons ($n = 1{,}000$) revealed that all approaches for determining P_{max} were correlated with one another though to varying degrees. The distribution of results from each method for calculating P_{max} calculation is illustrated in Figure 3 and described in Table 1. Relationships between each method are described in Table 2 and displayed as scatter plots in Figure 4. Across degrees of model fit, the observed P_{max} approach consistently provided more widely distributed values than the other approaches, and these ranges are described in Table 1. This approach reliably produced results that ranged much lower and much higher than other methods, see Figure 3. Consistent with the shared mathematical basis for the derivative and analytic approaches, the results from both approaches were perfectly correlated ($r = 1$). Similarly, the approximate method provided results that were highly correlated with both the derivative and analytic

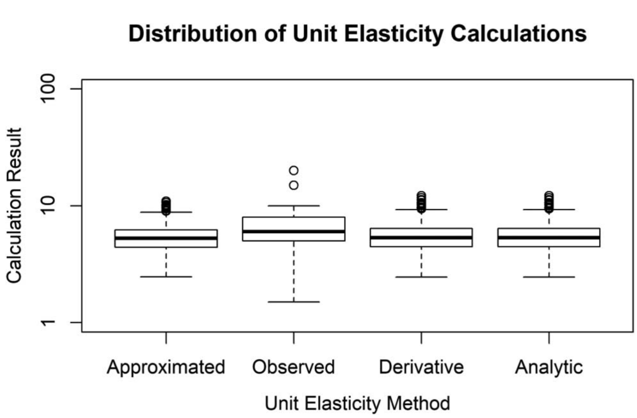

Figure 3. Box plot and unit elasticity distribution for simulated series. This figure illustrates the range of values the range of P_{max} values resulting from each type of calculation. The information provided here highlights substantial variability in the observed method, minimal variability in the approximate method, and exact correspondence between derivative and analytic methods.

methods ($rs = 0.99$). In contrast, the observed approach provided results that were not as strongly correlated overall with the approximate ($r = .29$), derivative ($r = .28$), or analytic P_{max} ($r = .28$). The results provided by the observed method were more highly correlated with the approximate ($r = .42$), derivative ($r = .42$), or analytic P_{max} ($r = .42$) in cases with better model fit (i.e., $R^2 > .9$). These relationships are more thoroughly described in Table 2 and illustrated in Figure 4.

Relationships between individual P_{max} calculations and various other indicators were evaluated using data from Kaplan and Reed (2018). Of the 1,104 participants completing the APT, 960 participants demonstrated indicators of systematic consumption (Stein et al., 2015) and were included in the final analysis. In these cases, nonlinear modeling converged in 874 of 960 cases (91.04%) and each calculation of P_{max} was evaluated against other related indicators of substance use derived from the Daily Drinking Questionnaire. For the empirical approach, Spearman correlations were nonsignificant between the observed P_{max} and number of reported monthly binges ($r = .00$; $p = .89$), total drinks ($r = -.01$; $p = .84$), and hours spent drinking ($r = .01$; $p = .78$). For all other P_{max} methods (i.e., approximate, derivative, and analytic) correlations and levels of significance were identical to the fifth decimal place. Specifically, Spearman correlations were nonsignificant between these measures of P_{max} and number of reported monthly binges ($rs = .03$; $ps = .33$), total drinks ($rs = .04$; $ps = .27$), and hours spent drinking ($rs = .05$; $ps = .15$). This comparison was conducted not to establish significant relations between P_{max} and substance use indicators, but to demonstrate the overall correspondence between the various approaches to calculating P_{max} and their relations with drinking measures. These results suggest that the approximate, derivative, and analytic methods of obtaining P_{max} may be interchangeable insofar as using them in nonparametric statistical tests (i.e., relative ranking is unaffected).

Discussion

The results from applied behavioral economic studies of demand are increasingly used as evidence to support various initiatives, such as investigating mechanisms related to substance abuse, as well as public policy (Hursh & Roma, 2013). However, the presence of varying approaches for calculating P_{max} naturally introduces some degree of error when determining prices that exist in the inelastic and elastic ranges. This is an area in need of precision and reliability, as both clinical and policy decisions may be directly or indirectly informed by the elasticity of demand for some good (e.g., nicotine, alcohol) or reinforcer (e.g., behavior functions, incentives). Precision is paramount here, as even small levels of variability could result in negative effects for clinical applications (e.g., poorer treatment outcomes), organizational-level decisions (e.g., employee attrition, ineffective incentive systems), and policy-level decisions alike (e.g., ineffective policy, limited replicability, poor use of taxpayer funding). Given that

[5] All elements of this report are provided under the GNU General Public License, Version 3.0, by Shawn P. Gilroy. The source code necessary to generate these simulations as well as perform each of P_{max} calculations is provided on the corresponding author's GitHub account in the repository named P_{max} Evaluation. Unless stated otherwise, all source code was written and executed within the R statistical program. These resources can be found at https://github.com/miyamot0/PmaxEvaluation.

Table 1
Distribution of Unit Elasticity Estimates in Simulated Data Series

Measure	Percentile				
	0	25	50	75	100
Overall distribution (N = 1,000)[a]					
Analytic	2.45655	4.46924	5.34454	6.39693	12.20367
Derivative	2.45655	4.46924	5.34454	6.39693	12.20367
Approximate	2.47089	4.4321	5.27383	6.21326	10.97175
Observed	1.5	5	6	8	20
Series with $R^2 > .9$ (N = 100)[b]					
Analytic	3.39425	4.43719	5.33661	6.17661	9.51373
Derivative	3.39425	4.43719	5.33661	6.17661	9.51373
Approximate	3.41699	4.42331	5.27033	6.09176	8.29142
Observed	2.5	5	6.5	8	20

Note. Quantile distributions for each of the unit elasticity methods.
[a] Overall R^2 for all series (M = .85, SD = .03). [b] Series with $R^2 > .9$ (M = .92, SD = .01).

there is a need to precisely and reliably determine P_{max}, this report was designed to answer the following questions: (a) To what degrees do the observed, approximate, derivative, and analytic methods for calculating P_{max} correspond with one another? (b) To what degree does the analytic method correspond with the derivative method of calculating P_{max}? (c) To what degree do results from the P_{max} methods correlate with related behavioral indicators?

Based on results from this study, all methods for calculating of P_{max} appear to be correlated with one another, though to varying degrees. That is, the approximate P_{max} was very closely related to the derivative P_{max} and the analytic P_{max} was identical to the derivative P_{max}. A 1:1 match between the derivative and analytic P_{max} was expected given the mathematical basis from which the analytic method was derived. However, the observed P_{max} method was less strongly correlated with the other calculations of P_{max} and this difference likely exists for several reasons. First, the observed P_{max} can only ever take the value of a unit price that was directly sampled. Given that this is an empirical measure, the observed P_{max} is inevitably constrained to a limited range of possible values. That is, this measure offers far less flexibility than other P_{max} methods. Second, prices used in purchase tasks are rarely equidistant (Kaplan, Foster, et al., 2018) and adjacent prices are likely to increment across several orders (e.g., 10 to 20, 50 to 100, 100 to 1,000). That is, prices are more densely sampled in the lower ranges and less densely sampled in the upper ranges. As a result, precision is more likely to decrease with a higher observed P_{max} simply because the distance between potential values is likely to grow at a near exponential rate. This is apparent in Table 1, as the discrepancy between the observed P_{max} and other methods appeared to grow larger with greater P_{max} values. To a much lesser degree, the approximate P_{max} consistently diverged from the derivative and analytic P_{max} as well. As indicated in Table 1, overall differences between the derivative/analytic and approximate methods became larger along with larger P_{max} values. As such, the approximate method appeared to introduce differential levels of error into the calculation of P_{max} as well.

Table 2
Correlation Matrix of Point of Unit Elasticity Methods in Simulated Data Series

Measure	Approximate	Derivative	Observed	Analytic
Overall (N = 1,000)				
Analytic	1	.99251	.29050	.99251
Derivative	—	—	.28274	1
Approximate	—	—	—	.28274
Observed	—	—	—	—
Series with $R^2 > .9$ (N = 100)				
Analytic	—	.99498	.4285	.99498
Derivative	—	—	.4210	1
Approximate	—	—	—	.4210
Observed	—	—	—	1

Note. Pearson correlation matrix for each of the four unit elasticity methods.

While providing identical results, the analytic P_{max} method offers several advantages over the computationally intensive derivative method. First, the analytic method is much simpler and can be performed by researchers without programming skills. For example, this method can be accommodated in customized spreadsheet software or even on websites. As an example of the simplicity, the analytic method has been adapted for use in a web-based calculator hosted by Shawn P. Gilroy.[6] In this way, researchers without programming skills can easily access a method for calculating an exact form of P_{max}. Further, this code can also be easily incorporated into existing behavioral economic software (Gilroy, Kaplan, Reed, Koffarnus, & Hantula, 2018; Kaplan, Gilroy, et al., 2018). Second, the analytic method also obviates the need for optimization and the potential for issues associated with it (e.g., poor starting values, approximations of Jacobian/Hessian). That is, the derivative method would require reasonable starting values to iteratively search for P_{max} (i.e., slope of -1), similar to parameter estimation. Third, improving the accessibility of this newer and exact form of P_{max} (relative to the derivative method) serves to limit, if not eliminate, the inherent variability introduced when relying on approximate or observed P_{max} values. That is, research-

[6] A web-based tool has been developed to support the use of analytic P_{max} and requires only the fitted model parameters. This tool is based on the GNU Scientific Library, accurate with study methods to the eighth decimal place, and provided at the following location: http://www.smallnstats.com/index.php?page=PMAX.

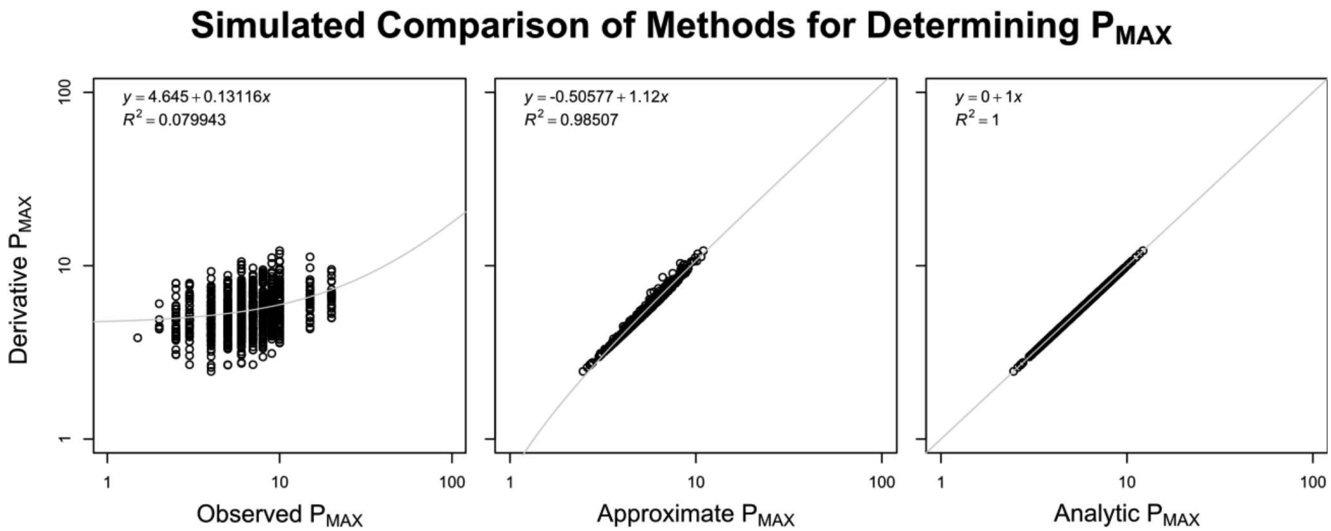

Figure 4. Comparison of exact and approximated P_{max} methods for simulated series. This figure illustrates relationships between methods for calculating P_{max}. The observed method, constrained to prices directly measured, varied substantially from methods that directly evaluated the slope demand curve. In contrast, the approximate method provided a consistent approximation of P_{max}, and the analytic method matched exactly with the derivative method.

ers may calculate the analytic P_{max} just as easily as they would calculate an approximate P_{max}, requiring only the Q_O, α, and k values. Even more, this approach effectively eliminates the error associated with relying on empirical or approximate calculations of P_{max} and the associated challenges with varying k values. In sum, this novel analytic approach consistently resulted in P_{max} values identical to the derivative, or exact, P_{max} and this method can be easily and reliably used in various behavioral economic applications.

Limitations

Although the methods introduced here provide one means of reducing sources of error in assessing elasticity, we note that the manner of calculating P_{max} is only one of several sources of variability. For example, the nature of purchase tasks and how consumption data are collected naturally influence the analyses that follow. That is, factors such as the range of prices sampled, the step sizes between these prices, and characteristics of individual prices (e.g., left-digit effects) can each affect analyses of demand. Even further, these novel methods are only an improvement insofar as the exponential or exponentiated models of demand effectively represent the data being modeled.

Notwithstanding that the results from computer simulation and real-world data are encouraging and support the analytic approach as a "drop-in" replacement for exact (i.e., derivative) P_{max} methods, additional replication is necessary for several reasons. First, additional study is necessary to understand the relative benefits of this novel approach over other methods for evaluating elasticity. For example, there may be situations in which differences between P_{max} approaches may be so minor that any potential differences would affect neither results from statistical comparisons between groups nor the inferences drawn. As such, in certain cases, the earlier approaches may introduce such low levels of error that subsequent analyses are not affected. Second, further study is necessary to determine how the limits imposed on k values in this newer method would impact real-world use. The issue of a hard lower limit may ultimately be a nonissue, as it is more often suggested to err on the side of larger k values by adding a small constant (i.e., 0.5; Kaplan, Foster, et al., 2018) and because k values less than 1 would suggest that consumption did not change even one $\log_{10}$ unit (i.e., a slope of -1 was not even observed). To this effect, further research on how to most effectively identify proper k values is necessary and the limits imposed by this method (i.e., W_0 and $-e^{-1}$) may be prudent as an absolute lower limit moving forward. However, the fitness of this guideline is an empirical question to be asked along with future replication with further real-world and simulated usage.

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